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# General properties of monochromatic optical beams

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**Abstract**

Using the idea of the angular spectral plane-wave expansion all the basic parameters of monochromatic optical beams in general were obtained, previously known by the Gaussian beams. The concept of a large-scale beam angle is introduced. In addition to the geometric phase shift, the interference nature of phase shift in beams was identified.
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## 1. Introduction

Representations of the properties of optical beams emitted by lasers are an important part of laser physics. To date, however, they are usually limited to the idea of Gaussian beams. In theoretical papers, even the idea of plane waves is used. In many cases, they are justified, but, strictly speaking, only valid for an unlimited space. The actual field propagation in a beam is always limited, primarily by transverse dimensions of the emitting laser active medium, and then by various elements in the beam path. Nevertheless, by default it is considered that laser radiation has parameters of Gaussian beams: beam waist with its radius, boundary between the near and far zone, divergence in the far zone. These, in fact intuitive, representations require at least a qualitative analysis of the properties of light beams in general. The proposed work is devoted to this matter.

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## 2. Theory

To analyze the basic properties of beams, it would be preferable to limit the problem by the conditions specific to laser radiation: high monochromaticity and directionality. Typically, the propagation medium contains no free electric charges. Then two basic Maxwell's equations are self-contained for monochromatic electromagnetic waves. The other two are performed automatically [Landau, Lifshitz (1992)]. In the optical frequency range the magnetic properties of the medium can be disregarded [Landau, Lifshitz (1992)]. Under these conditions, the magnetic field is usually excluded from consideration and the wave equation is derived for the electric field $\vec{E}$ [Born, Wolf (1973)]:

$$\nabla^2 \vec{E} - \frac{\varepsilon}{c^2} \ddot{\vec{E}} = 0 \tag{1}$$

The inhomogeneity and anisotropy of the medium are known to lead to optical phenomena that cause a change in parameters of light beams and in the direction of their propagation. It is clear that they should be treated independently and not directly related to the original parameters of emitted beams. The same applies to the absorption or amplification of light. Therefore, we can consider the dielectric constant of the medium $\varepsilon$ as a real scalar value determining the refractive index $\sqrt{\varepsilon}$ and is independent on the coordinates $\vec{r}$ .

Any solution of the homogeneous equation (1) can be represented by a superposition of its fundamental solutions. For example, the use of spherical waves in the Fresnel-Kirchhoff integral is the basis of the diffraction theory. Fourier optics is based on the use of plane waves. Therefore, each transverse field component $E$ of the light beam with a frequency $\omega$ can be represented by an integral over solid angles $\Omega$ in space of the wave vectors $\vec{k}$ with a radius $k = \sqrt{\varepsilon}\, \omega/c$ :

$$E = \frac{A}{2\pi k^2} \exp(-i\omega t) \int_{\Omega} F(\vec{k}) \exp(i\vec{k}\vec{r}) d\vec{k} \tag{2}$$

The function $F$ with the normalization: $\int_{\Omega} F(\vec{k}) d\vec{k} = 2\pi k^2$ describes the dimensionless propagation of complex amplitudes of plane waves forming the beam over radial $\theta$ and azimuthal $\varphi$ angles $\vec{k}$ with respect to the axis of its propagation $z$ , i.e. represents a two-dimensional angular spectrum of the beam. The value $A$ is a complex amplitude of the field at the origin: $\vec{r} = 0$ . To be specific, we consider axially symmetric beams. For them the $F$ is the even function $\theta$ and the periodic function $\varphi$ with a period $2\pi/n$ , where $n$ is an integer. It can be expanded in a Fourier series, and we consider functions in the form: $F = f(\theta^2) \exp(in\varphi)$ . In cylindrical coordinates $\vec{r} = (z, \rho, \psi)$ , integrating in (2) over $\varphi$ , we get:

$$E = A \exp(-i\omega t)(i)^n \exp(in\psi) \int_0^{\pi/2} f(\theta^2) J_n(k\rho \sin\theta) \exp(ikz\cos\theta) \sin\theta d\theta \tag{3}$$

where $J_n$ is the Bessel function of order $n$ . To analyze the general properties of beams in free space, it is sufficient to consider the fundamental mode beam: $n = 0$ , the $f$ of which has a single maximum in the direction of dominant propagation of energy: $\theta = 0$ . With a narrow beam the function $f$ can be characterized by a half-width of the angular spectrum $\theta_m << 1$, having a sense of the large-scale angle of plane wave propagation. In (3) we may take: $\sin\theta \cong \theta$, $\cos\theta \cong 1-\theta^2/2$, the upper limit is regarded as the infinite one, and we get:

$$E = A\exp\left(i(kz-\omega t)\right)u(\vec{r}),\; u = \int_0^\infty f\left(\frac{\theta^2}{\theta_m^2}\right)J_0(k\rho\theta)\exp\left\{-ik\theta^2 z/2\right\}\theta d\theta \qquad (4)$$

With the normalization: $\int_0^\infty f\left(\frac{\theta^2}{\theta_m^2}\right)\theta d\theta = 1$.

In general, $f$ is a complex value. However, taking it for the real one from (4) it can be seen that with the longitudinal coordinate $z = 0$ the value $u$ is real for any radial coordinate $\rho$. This means that the phase across all this plane is the same. Hence, the beam wavefront is flat. Referring a sign to the value $A$, we can consider $f > 0$.

It may be noted that with the real $f$ the value $u$ at $\rho = 0$, considered as a function of the complex variable $z$, belongs to the same class as the dielectric susceptibility of the medium as a function of the complex frequency[Landau, Lifshitz (1992)]. The only difference is that $u(z)$ is analytic not in the upper, but in the lower complex half-plane. Consequently, for example, the Kramers-Kronig relations with the sign change are valid for its real $u'$ and imaginary $u''$ parts [Landau, Lifshitz (1992)]. For the analysis carried out here, it is important to note that in accordance with (4) $u^*(z) = u(-z)$, and therefore the real part $u'(z)$ of the value $u = u' + iu''$ is even with a maximum at $z = 0$ and the imaginary one is odd, negative at $z > 0$. When: $|z| << 2/(k\theta_m^2)$, which determines the value $u$ is $u'$. It decreases slightly with the increase of $|z|$. This means that the beam diverges slightly the same way in both directions of $z$. This can be seen on the example of the Gaussian distribution $f = \left(2/\theta_m^2\right)\exp\left(-\theta^2/\theta_m^2\right)$ at $\rho = 0$:

$$u' = \frac{1}{1+a^2z^2},\; u'' = \frac{az}{1+a^2z^2},\; a = \frac{1}{2}k\theta_m^2,\; |u| = \frac{1}{\sqrt{1+a^2z^2}}, \qquad (5)$$

and at $|z| << 2/(k\theta_m^2)$: $|u| \approx 1-(k\theta_m^2)z^2/8$.

Thus, the choice of the real $f$ is the choice of the center of coordinates in the beam waist.

As can be seen from (4), the amplitude in the waist is significantly reduced if $\rho$ reaches the value $w_0$, that the first half-wave $J_0$ fits the range of zero to $\theta_m$ : $kw_0\theta_m/2 \cong 1$. Accordingly, the radius of the waist can be considered as the value $w_0 = 2/k\theta_m$. In rarefied media we can neglect the difference of the refractive index from unity and consider: $k = 2\pi/\lambda$, where $\lambda$ is the wavelength. This shows that in the near beam zone from the waist the large-scale angle plays the role of the diffraction angle at the waist: $\theta_m = \lambda/\pi w_0$.

In the far zone at $z \to \pm\infty$, $u' \to 0$ and $u'' \to 0$, but because of their parities: $u' \sim z^{-2}$, $u'' \sim -z^{-1}$. And the incoming wave phase: $\mathrm{arctg}\left(u''/u'\right) \to \mp\pi/2$, additional to the geometrical $kz$, can be seen as the result of interference of plane waves forming the beam. It is similar to the additional phase that appears after the beam has passed through a multiple beam interferometer. Since in the far zone, $u''$ appears to be dominant, and in (4) it is defined by $\sin(k\theta^2 z/2)$, the radius $w$ of the beam in the far zone can be estimated from the condition that at $\rho = w$ the oscillations over $\theta$ of the functions $\sin(k\theta^2 z/2)$ and $J_0(k\rho\theta)$, having the form of a damped cosine, in the range of zero to $\theta_m$ turned out to be in qualitatively quadrature relative to each other (similarly $\cos x$ and $\sin x$, in the product of which the number of oscillations is doubled and there is the factor 1/2). This significantly reduces the value $u$. Hence, $kw\theta_m = kw\theta_m^2|z|$ and we can assume: $w = \theta_m|z|$. The beam takes the form of a limited spherical wave. Here the large-scale angle plays the role of the beam divergence angle.

The boundary $z = p$ between the far and near zone can be determined from the condition of equality $u'$ and $|u''|$ at $\rho = 0$. For the step-function $f$ equal to constant value in the range $\theta$ of zero to $\theta_m$ and equal to zero at $\theta > \theta_m$, from (4) it is easy to obtain $k\theta_m^2 p/2 = \pi/4$. It is clear that for the smooth function $f$, this value will be slightly larger. Therefore, in general, we can assume approximately: $k\theta_m^2 p/2 = 1$. It is easy to see that for the Gaussian function, this equality is true (5). Hence we obtain: $p = \dfrac{2}{k\theta_m^2}$, or from the waist radius $p = \pi w_0^2/\lambda$ – the value known also as the wave beam parameter.

It is useful to accompany the obtained results with the specific example. We consider the Gaussian distribution of plane waves forming the beam in the radial angles: $f(\dfrac{\theta^2}{\theta_m^2}) = \dfrac{2}{\theta_m^2}\exp(-\dfrac{\theta^2}{\theta_m^2})$. The integration in (4) is carried out by the representation of $J_0$ as power series, term-by-term integration and the use of the representation of the exponential as power series. The result is a well-known expression for the Gaussian beam (e.g. [Oraevsky (1988)]):

$$u = \frac{1}{\sqrt{1+\left(\dfrac{z}{p}\right)^2}}\exp\left[-\left(\frac{\rho}{w}\right)^2 + ik\frac{\rho^2}{2R} - iarctg\left(\frac{z}{p}\right)\right], \text{ where}$$

$$p = \frac{2}{k\theta_m^2},\ w = w_0\sqrt{1+\left(\frac{z}{p}\right)^2},\ w_0 = \frac{2}{k\theta_m},\ R = z + \frac{p^2}{z}$$

Here all the beam parameters: wave parameter $p$, waist radius $w_0$, radius $w(z)$ and radius of the wavefront $R$ are defined by the large-scale angle.

## 3. Conclusion

As a result, using the idea of the angular spectra all the basic parameters of monochromatic optical beams in general were obtained, previously known by the Gaussian beams. The concept of a large-scale beam angle is introduced, which determines all its parameters and appears in the form of a diffraction angle at the waist and a divergence angle in the far zone of the beam. In addition to the geometric phase shift, the interference nature of phase shift in beams was identified. It is hoped that these results extend the understanding of propagation of light beams.

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